Activity 1

System of linear equations

Objective

To obtain the conditions for consistency of a system of linear equations in two variables by graphical method.

Materials required

3 graph papers,
ruler,
pencil.

Pre-requisite knowledge

Plotting of points on a graph paper

Procedure

1. Take the first pair of linear equations in two variables of the form

$$a_1x + b_1y + c_1 = 0$$
$$a_2x + b_2y + c_2 = 0$$

e.g. $2x - 3y = 3$
$3x - 4y = 5$

2. Obtain a table of ordered pairs (x, y), which satisfy the given equation. Find at least three such pairs for each equation.

e.g. For $2x - 3y = 3$

x	0	3	6
y	-1	1	3

For $3x - 4y = 5$

x	- 1	- 5	7
y	- 2	- 5	4

3. Plot the graphs for the two equations on the graph paper as shown in Fig 1(a).

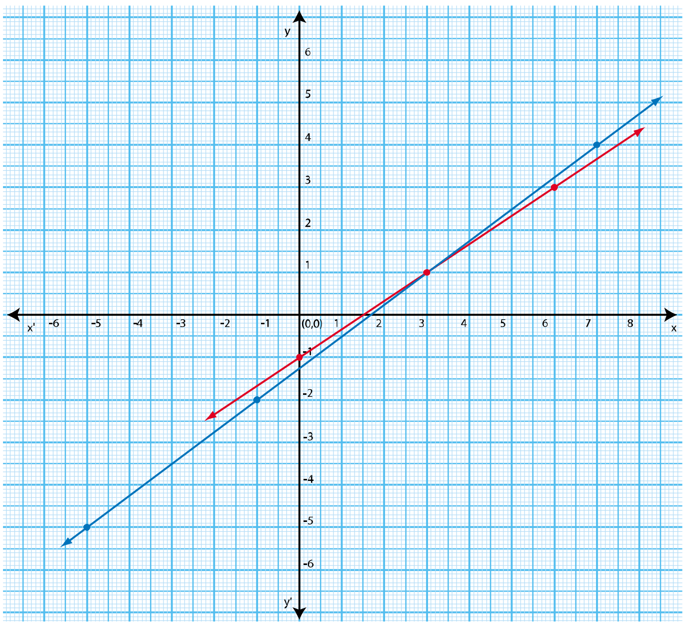


Fig 1(a)

4. Observe if the lines are intersecting, parallel or coincident and note the following:

$\frac{a_1}{a_2} =$ $\frac{b_1}{b_2} =$ $\frac{c_1}{c_2} =$

5. Take the second pair of linear equations in two variables,
   e.g. $6x + 10y = 4$, $3x + 5y = -11$
6. Repeat the steps from 2 to 4.
7. Take the third pair of linear equations in two variables
   e.g. $x - 2y = 5$, $2x - 4y = 10$
8. Repeat the steps from 2 to 4.
9. Fill in the following observation table

Type of lines	$\frac{a_1}{a_2}$	$\frac{b_1}{b_2}$	$\frac{c_1}{c_2}$
Intersecting			
Parallel			
Coincident			

10. Obtain the condition for two lines to be intersecting, parallel or coincident from the observation table by comparing the values of $\frac{a_1}{a_2}$, $\frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$.

Observations

The student will observe that for intersecting lines $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$,

for parallel lines $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ and for coincident lines $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$.

Learning outcomes

The student will learn that some pairs of linear equations in two variables have a unique solution (intersecting lines); some have infinitely many solutions (coincident lines) and some have no solution (parallel lines).

Remarks

1. The teacher will explain that when a system of linear equations has solution (whether unique or not), the system is said to be consistent; when the system of linear equations has no solution, it is said to be inconsistent.

2. The teacher may consider additional examples in which some of the co-efficients are zero.

Activity 2

Arithmetic Progression - I

Materials required

coloured paper,
pair of scissors,
geometry box,
fevicol,
sketch pens,
one squared paper.

Objective

To verify that the given sequence is an arithmetic progression by paper cutting and pasting method.

Pre-requisite knowledge

1. Definition of an arithmetic progression

Procedure

1. Take a given sequence of numbers say a_1, a_2, a_3 ....
2. Cut a rectangular strip from a coloured paper of width k = 1 cm (say) and length a_1 cm.
3. Repeat this procedure by cutting rectangular strips of the same width k = 1cm and lengths a_2, a_3, a_4, ... cm.
4. Take 1 cm squared paper and paste the rectangular strips adjacent to each other in order.

A] Let the sequence be 1, 4, 7, 10, ...

Take strips of lengths 1 cm, 4 cm, 7 cm and 10 cm, all of the same width say 1 cm. Arrange the strips in order as shown in Fig 2(a). Observe that the adjoining strips have a common difference in heights. (In this example it is 3 cm.)

B] Let another sequence be 1, 4, 6, 9, …

Take strips of lengths 1 cm, 4 cm, 6 cm and 9 cm all of the same width say 1 cm. Arrange them in an order as shown in Fig 2(b). Observe that in this case the adjoining strips do not have the same difference in heights.

So, from the figures, it is observed that if the given sequence is an arithmetic progression, a ladder is formed in which the difference between the heights of adjoining steps is constant. If the sequence is not an arithmetic progression, a ladder is formed in which the difference between adjoining steps is not constant.

Learning outcome

Students learn the meaning of an arithmetic progression by relating it to an activity that involves visualization.

Remark

The teacher may point out that in this activity taking width of the strips to be constant is not essential but convenient for visual simplicity of the ladder.

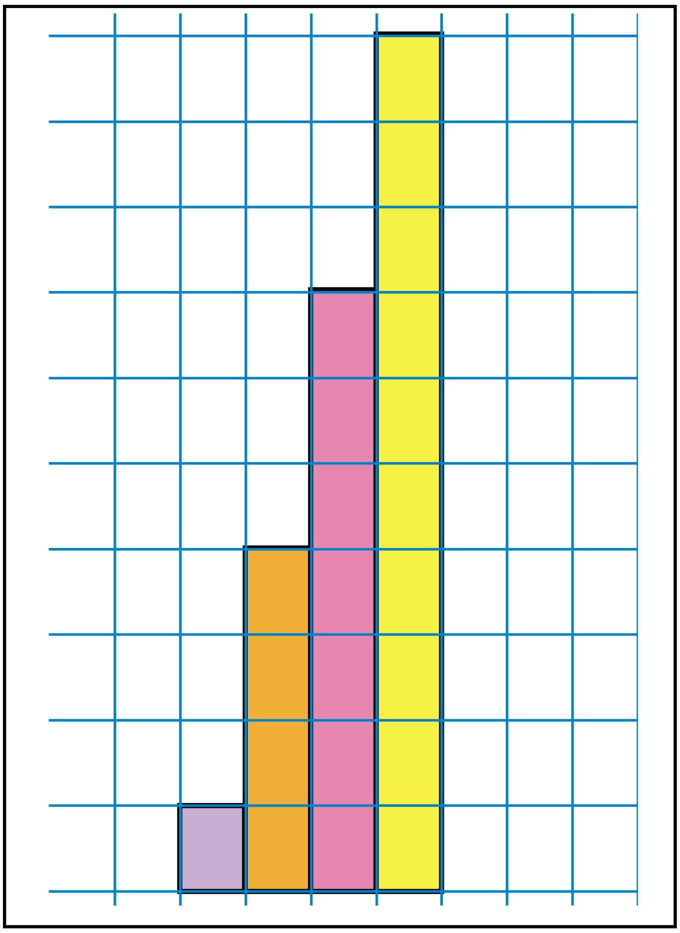


Fig 2(a)

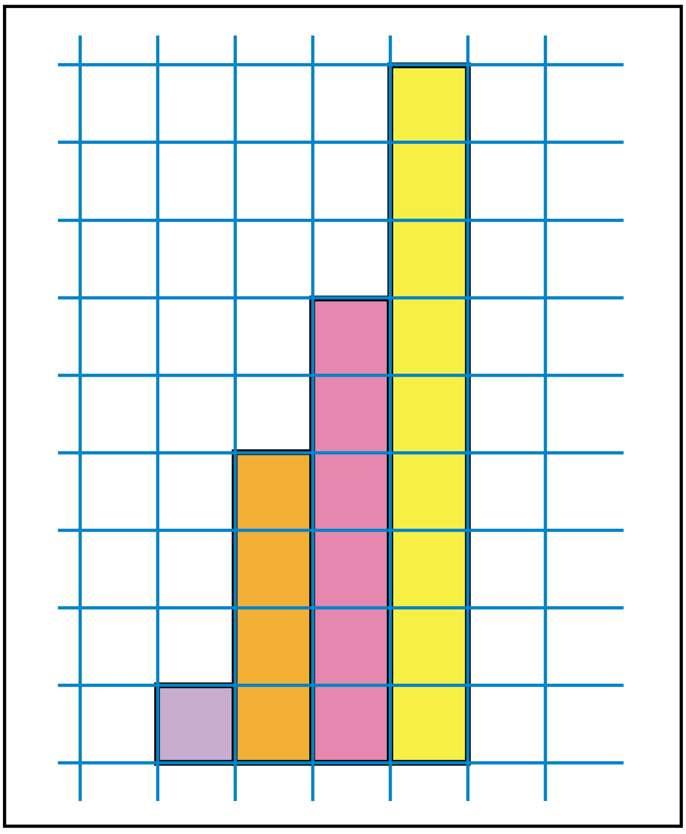


Fig 2(b)

Activity 3

Arithmetic Progression - II

Materials required

chart paper,
sketch pens,
geometry box,
squared paper.

Objective

To verify that the sum of first n natural numbers is $n(n+1)/2$, i.e. $\Sigma n = n(n+1)/2$, by graphical method.

Pre-requisite knowledge

1. Natural number system
2. Area of squares and rectangles

Procedure

Let us consider the sum of natural numbers say from 1 to 10, i.e. $1 + 2 + 3 + \ldots + 9 + 10$. Here $n = 10$ and $n + 1 = 11$.

1. Take a squared paper of size 10×11 squares and paste it on a chart paper.
2. On the left side vertical line, mark the squares by 1, 2, 3, … 10 and on the horizontal line, mark the squares by 1, 2, 3 …. 11.
3. With the help of sketch pen, shade rectangles of length equal to 1 cm, 2 cm, …, 10cm and of 1 cm width each.

Observations

The shaded area is one half of the whole area of the squared paper taken. To see this, cut the shaded portion and place it on the remaining part of the grid. The student will observe that it completely covers the grid.

Area of the whole squared paper is 10×11 cm ^{2}.
Area of the shaded portion is $(10 \times 11)/2$ cm^2.
This verifies that, for $n = 10$,

$$\Sigma n = n \times (n+1)/2$$

The same verification can be done for any other value of n.

Learning outcome

Students develop a geometrical intuition of the formula for the sum of natural numbers starting from one.

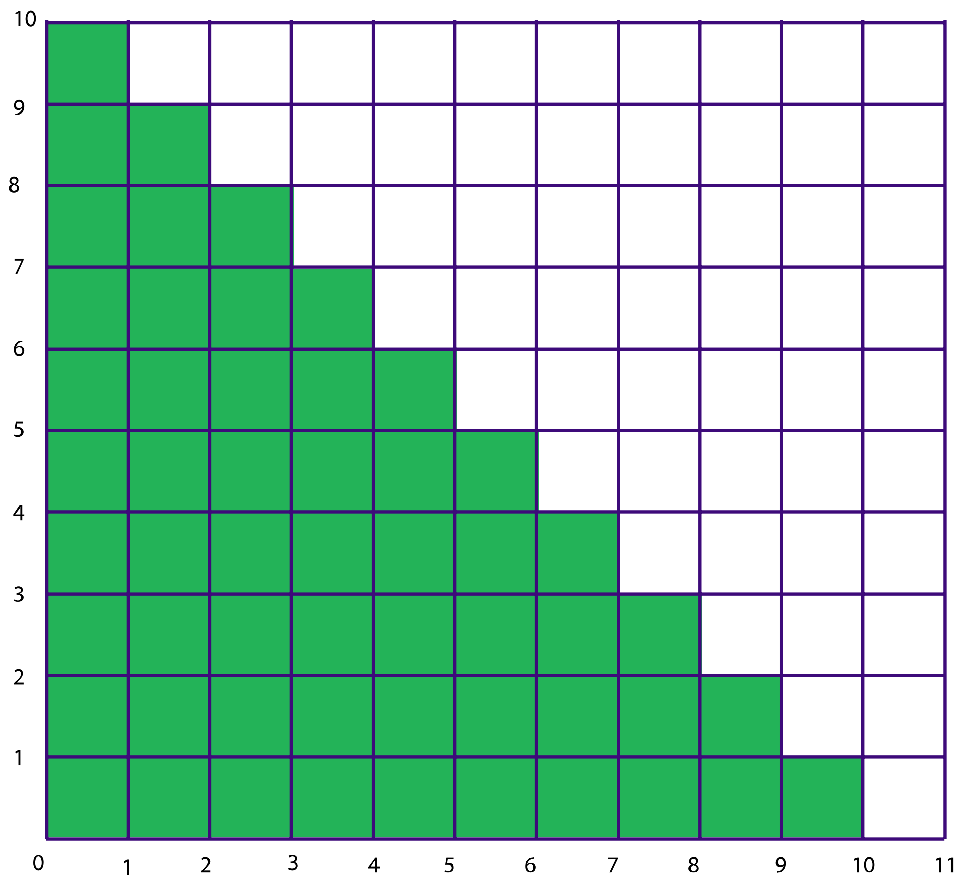


Fig 3

Activity 4

Basic Proportionality Theorem for a Triangle

Objective

To verify the Basic Proportionality Theorem using parallel line board and triangle cut-outs.

Materials required

coloured paper,
pair of scissors,
parallel line board,
ruler,
sketch pens.

> ***Basic Proportionality Theorem***
> *If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.*

Pre-requisite knowledge

Drawing parallel lines on a rectangular sheet of paper.

Procedure

1. Cut three different triangles from a coloured paper. Name them as ΔABC, ΔPQR and ΔDEF.
2. Take the parallel line board (a board on which parallel lines are drawn) as shown in Fig 4 (a). (Note: Students can make the parallel line board, using the techniques given in class IX laboratory manual.)
3. Place ΔABC on the board such that any one side of the triangle is placed on one of the lines of the board as shown in Fig 4 (b). (It would be preferable to place the triangle on the lowermost or uppermost line.)
4. Mark the points P_1, P_2, P_3, P_4 on ΔABC as shown in Fig 4(b). Join P_1P_2 and P_3P_4. $P_1P_2 \parallel BC$ and $P_3P_4 \parallel BC$
5. Note the following by measuring the lengths of the respective segments using a ruler.

Ratios	Value
$\frac{AP_1}{P_1B}$	
$\frac{AP_2}{P_2C}$	
$\frac{AP_3}{P_3B}$	
$\frac{AP_4}{P_4C}$	

6. Repeat the experiment for ΔDEF and ΔPQR.

Observations

1. Students will observe that

In Δ ABC $\quad \dfrac{AP_1}{P_1B} = \dfrac{AP_2}{P_2C}$

$$\dfrac{AP_3}{P_3B} = \dfrac{AP_4}{P_4C}$$

2. Students will note similar equalities for Δ DEF and Δ PQR.
3. Students will observe that in all the three triangles the Basic Proportionality Theorem is verified.

Learning outcome

Knowledge of the Basic Proportionality Theorem for a triangle will be reinforced through this activity.

Remark

The teacher will point out to the students to observe that $P_1P_2 \parallel BC$ and $P_3P_4 \parallel BC$ because segments P_1P_2 , P_3P_4 and BC are part of the lines parallel to each other.

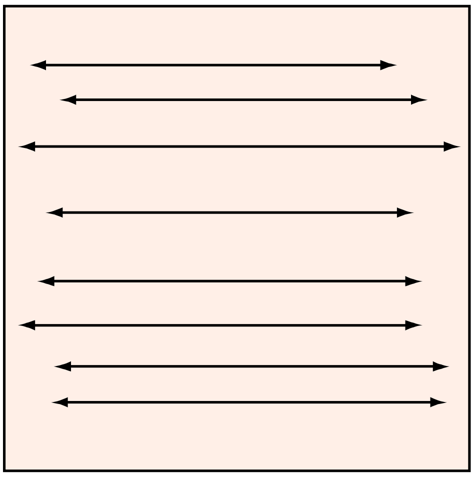


Fig 4(a)

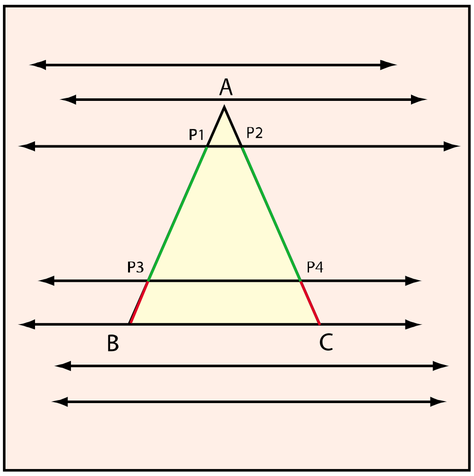


Fig 4(b)

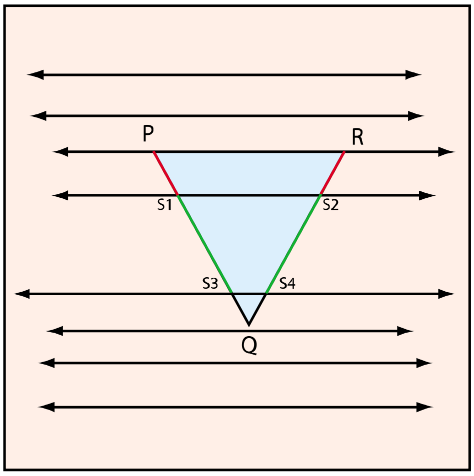


Fig 4(c)

Activity 5

Pythagoras theorem

Materials required

card board,
coloured pencils,
pair of scissors,
fevicol,
geometry box.

Objective

To verify the Pythagoras Theorem by the method of paper folding, cutting and pasting.

> ***Pythagoras Theorem***
> *The area of the square on the hypotenuse of a right angled triangle is equal to the sum of the areas of squares on the other two sides.*

Pre-requisite knowledge

1. Area of a square.
2. Construction of parallel lines and perpendicular bisectors.
3. Construction of a right angled triangle.

Procedure

1. Take a card board of size say 20 cm × 20 cm.
2. Cut any right angled triangle and paste it on the cardboard. Suppose its sides are *a*, *b* and *c*.
3. Cut a square of side *a* cm and place it along the side of length *a* cm of the right angled triangle.
4. Similarly cut squares of sides *b* cm and *c* cm and place them along the respective sides of the right angled triangle.
5. Label the diagram as shown in Fig 5(a).
6. Join BH and AI. These are two diagonals of the square ABIH. The two diagonals intersect each other at the point O.
7. Through O, draw RS || BC.
8. Draw PQ, the perpendicular bisector of RS, passing through O.
9. Now the square ABIH is divided in four quadrilaterals. Colour them as shown in Fig 5(a).
10. From the square ABIH cut the four quadrilaterals. Colour them and name them as shown in Fig 5(b).

Observations

The square ACGF and the four quadrilaterals cut from the square ABIH completely fill the square BCED. Thus the theorem is verified.

Learning Outcome

Students learn one more method of verifying Pythagoras theorem.

Remark

The teacher may point out that the activity only verifies Pythagoras theorem for the given triangle. Verification is different from a general mathematical proof.

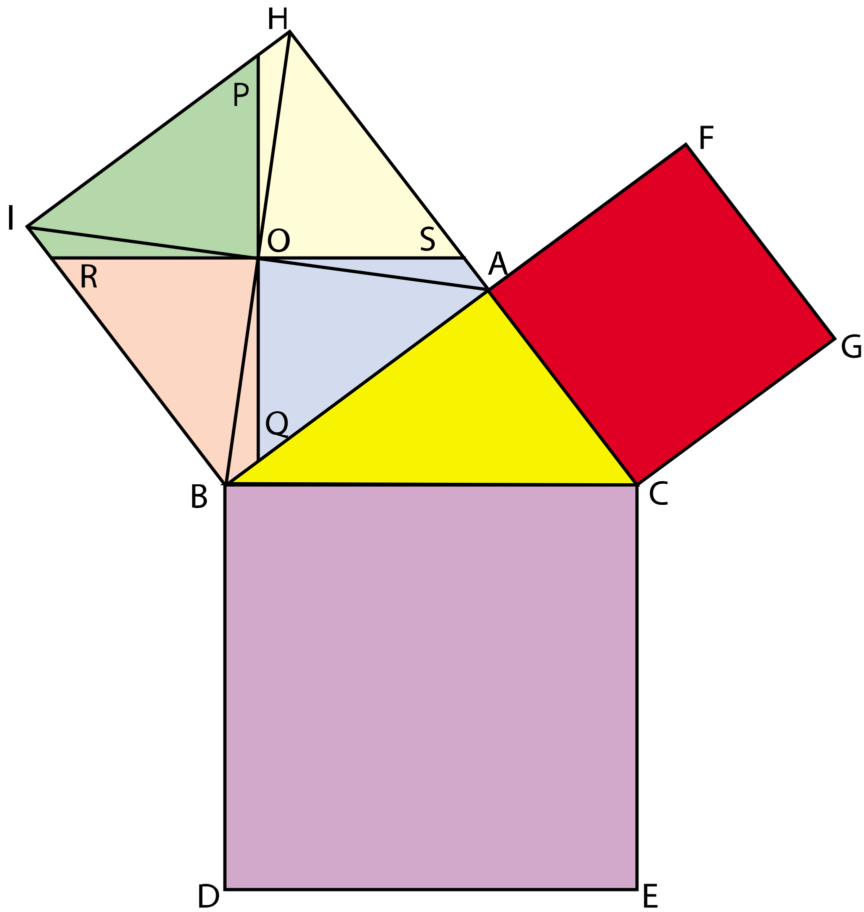


Fig 5(a)

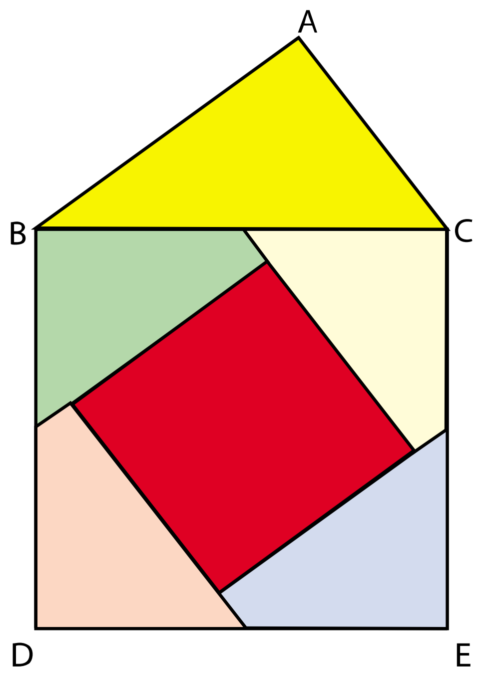


Fig 5(b)

Activity 6

Angle at the centre of a circle by an arc

Materials required

coloured papers,
a pair of scissors,
gum,
compass,
pencil,
ruler,
carbon paper or
tracing paper.

Objective

To verify that the angle subtended by an arc at the centre of a circle is twice the angle subtended by the same arc at any other point on the remaining part of the circle, using the method of paper cutting, pasting and folding.

Pre-requisite knowledge

Meaning of subtended angle by an arc.

Procedure

1. Draw a circle of any radius with centre O on a coloured sheet of paper and cut it.
2. Take a rectangular sheet of paper and paste the cutout on it. [Fig 6(a)]
3. Take two points A and B on the circle to obtain arc AB. [Fig 6(b)]
4. Form a crease joining OA and draw OA. [Fig 6(c)]
5. Form a crease joining OB and draw OB. [Fig 6(d)]
6. Arc AB subtends $\angle$AOB at the centre O of the circle. [Fig 6(e)]
7. Take a point P on the remaining part of the circle.
8. Form a crease joining AP and draw AP. [Fig 6(f)]
9. Form a crease joining BP and draw BP. [Fig 6(g)]
10. Arc AB subtends $\angle$APB at the point P on the remaining part of the circle. [Fig 6(h)]
11. Make two replicas of $\angle$APB using carbon paper or tracing paper. [Fig 6(i)]
12. Place two replicas of $\angle$APB adjacent to each other on $\angle$AOB. What do you observe? [Fig 6(j)]

Observations

1. Students will observe that two replicas of $\angle$APB completely cover $\angle$AOB.
2. $\angle AOB = 2\angle APB$

Learning outcome

Students become more familiar with the theorem (that is proved in classroom) through an activity.

Remarks

1. The teacher may ask the students to perform the activity for the cases where arc AB is a major arc or a semi circular arc.
2. The teacher should point out that the activity only verifies the theorem and is not a proof of the theorem.

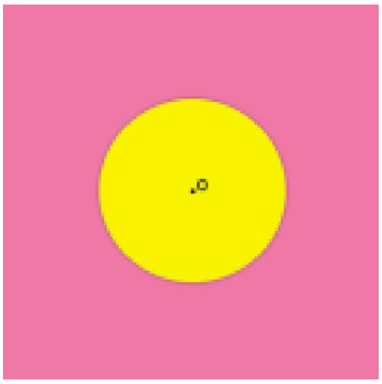


Fig 6(a)

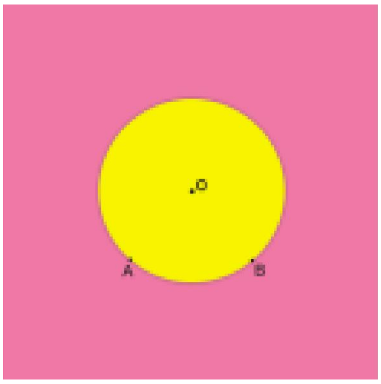


Fig 6(b)

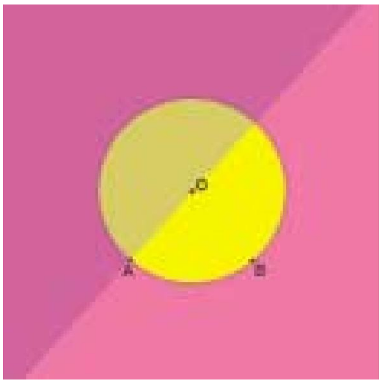


Fig 6(c)

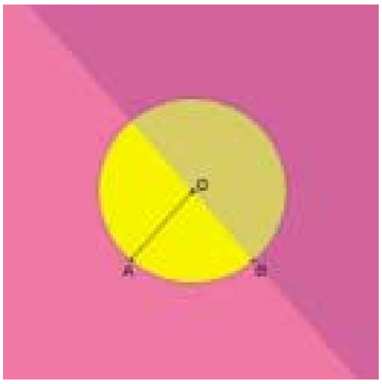


Fig 6(d)

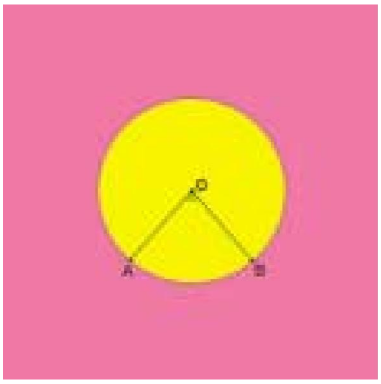


Fig 6(e)

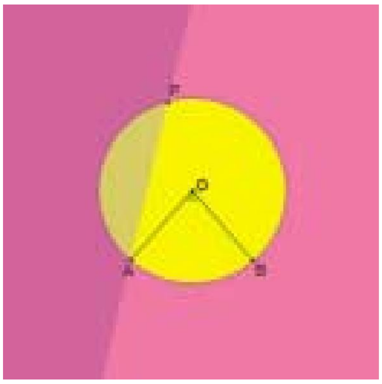


Fig 6(f)

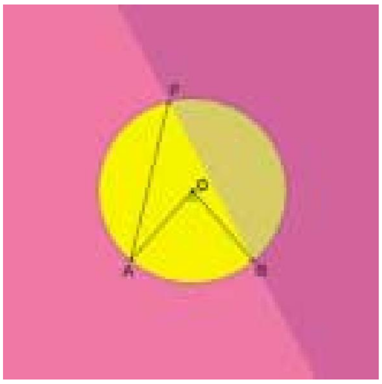


Fig 6(g)

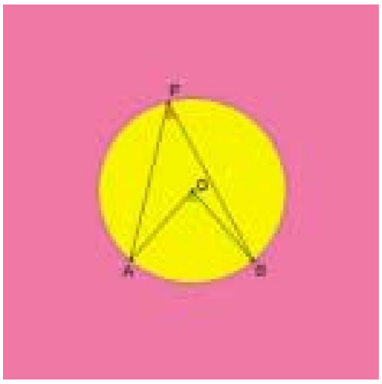


Fig 6(h)

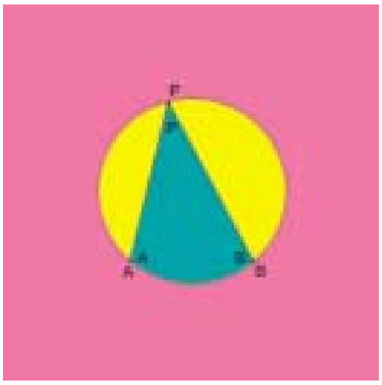


Fig 6(i)

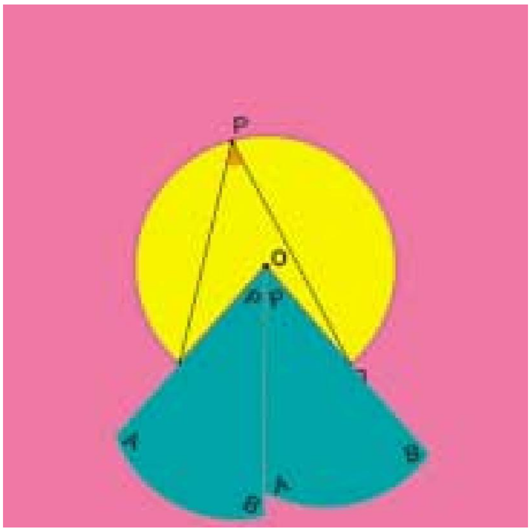


Fig 6(j)

Activity 7

Angles in the same segment

Materials required

coloured papers,
pair of scissors,
gum,
scale,
compass,
pencil,
carbon papers or
tracing papers.

Objective

To verify that the angles in the same segment of a circle are equal, using the method of paper cutting, pasting and folding.

Pre-requisite knowledge

Geometrical meaning of segment of a circle.

Procedure

1. Draw a circle of any radius with centre O and cut it.
2. Paste the cutout on a rectangular sheet of paper. [Fig 7(a)].
3. Fold the circle in any way such that a chord is made. Draw the line segment AB. [Fig 7(b)].
4. Take two points P and Q on the circle in the same segment. [Fig 7(c)].
5. Form a crease joining AP. Draw AP. [Fig 7(d)].
6. Form a crease joining BP. Draw BP. [Fig 7(e)].
7. $\angle$APB is formed in the major segment. [Fig 7(f)]
8. Form a crease joining AQ. Draw AQ. [Fig 7(g)]
9. Form a crease joining BQ. Draw BQ. [Fig 7(h)]
10. $\angle$APB and $\angle$AQB are formed in the major segment. [Fig 7(i)]
11. Make replicas of $\angle$APB and $\angle$AQB using carbon paper or tracing paper. [Fig 7(j)]
12. Place the cutout of $\angle$APB on the cutout of $\angle$AQB. What do you observe?

Observations

Students will observe that

1. $\angle$APB and $\angle$AQB are angles in the same segment.
2. $\angle$AQB covers $\angle$APB exactly. Therefore $\angle APB = \angle AQB$

Learning outcome

Students become more familiar with this theorem (proved in the classroom) through an activity.

Remarks

1. The teacher may ask the student to perform the activity by taking different points P and Q including those very close to points A and B.
2. The teacher may ask the student to perform the activity using point P in one segment and Q in the other segment and note that the angles in the segments are not equal, except in the case when the chord AB is a diameter of the circle. They will find that the two angles are supplementary angles.

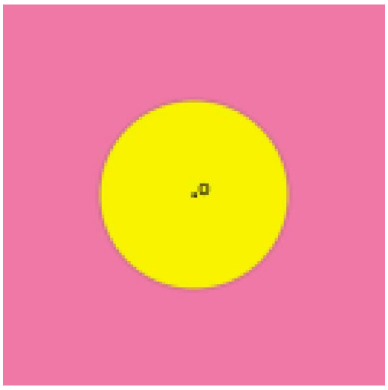


Fig 7(a)

Fig 7(b)

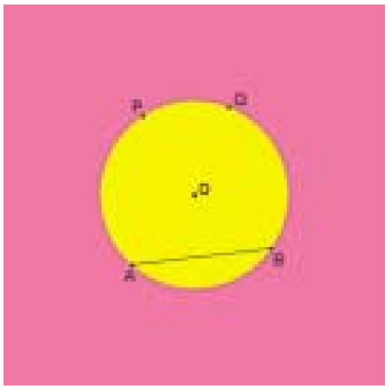


Fig 7(c)

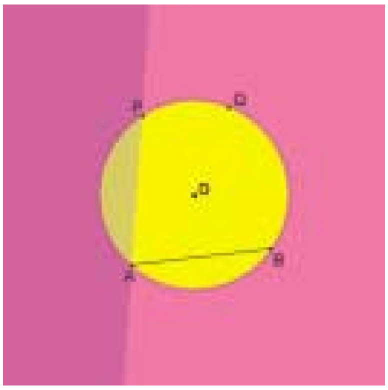


Fig 7(d)

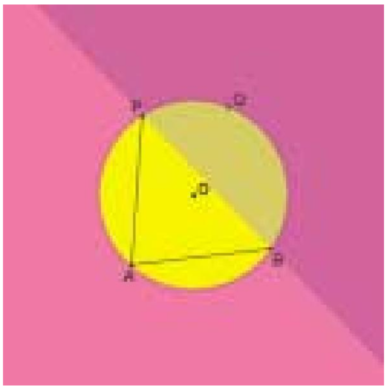


Fig 7(e)

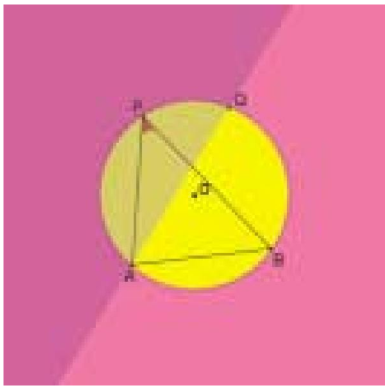


Fig 7(f)

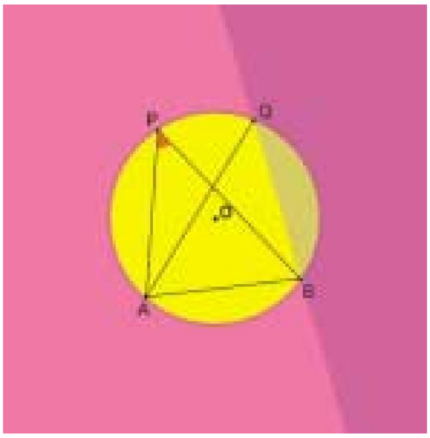


Fig 7(g)

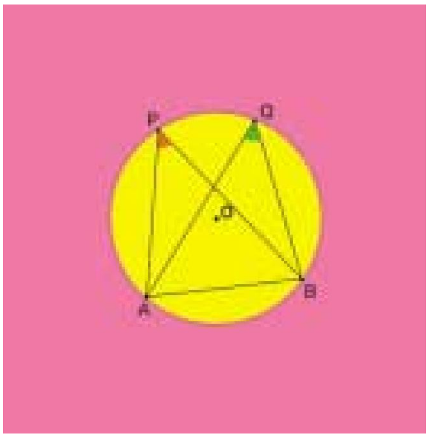


Fig 7(h)

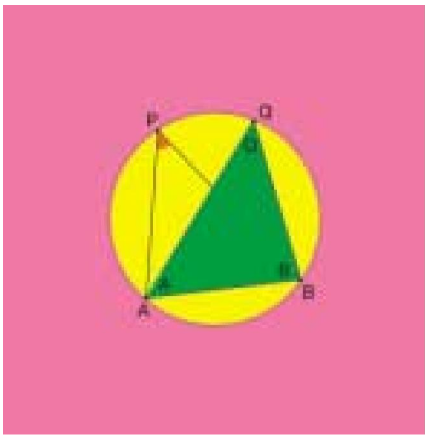


Fig 7(i)

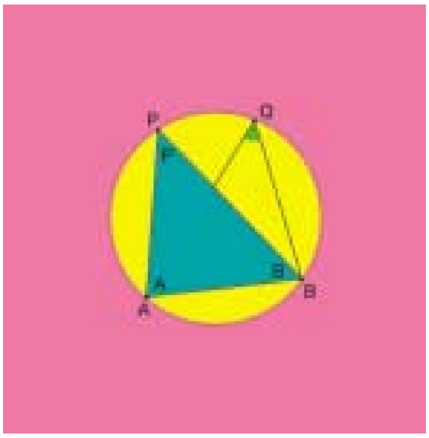


Fig 7(j)

Activity 8

Angle in a semicircle, major segment and minor segment

Materials required

coloured papers,
a pair of scissors,
gum,
scale,
compass,
pencil,
carbon paper or tracing paper,
cutout of right angled triangle.

Objective

To verify, using the mrthod of paper cutting, pasting and folding that

(a) the angle in a semicircle is a right angle,
(b) the angle in a major segment is acute,
(c) the angle in a minor segment is obtuse.

Pre-requisite knowledge

Concept of right angle, acute angle, obtuse angle, linear pair axiom.

Procedure for (a)

1. Draw a circle of any radius with centre O on a coloured sheet of a paper and cut it. [Fig 8a(a)]
2. Form a crease passing through the centre O of the circle. Diameter AB is obtained. Draw AB. [Fig 8a(b)]
3. Take a point P on the semicircle.
4. Form a crease joining AP. Draw AP. [Fig 8a(c)]
5. Form a crease joining BP. Draw BP. [Fig 8a(d)]
6. Make two replicas of $\angle APB$; call them $\angle A_1P_1B_1$ and $\angle A_2P_2B_2$. [Fig 8a(e)]
7. Place the two replicas adjacent to each other such that A_1P_1 and P_2B_2 coincide with each other as shown in Fig 8a(f).

Observations

Students will observe that the two line segments P_1A_1 and P_2B_2 lie on a straight line.

Therefore, $\angle A_2P_2B_2 + \angle B_1P_1A_1 = 180^0$

But $\angle A_2P_2B_2$ and $\angle B_1P_1A_1$ are replicas of $\angle APB$.

i.e. $2\angle APB = 180^0$

i.e. $\angle APB = 90^0$

Procedure for (b)

1. Draw a circle of any radius with centre O on a coloured sheet of paper and cut it.
2. Paste the cutout on a rectangular sheet of paper. [Fig 8b(a)]
3. Fold the circle in such a way that a chord AB is obtained. Draw AB. [Fig 8b(b)]
4. Take a point P in the major segment.
5. Form a crease joining AP. Draw AP. [Fig 8b(c)]
6. Form a crease joining BP. Draw BP. [Fig 8b(d)]
7. Make a replica of $\angle APB$. [Fig 8b(e)].
8. Place the replica of $\angle APB$ on a right angled Δ DEF such that BP falls on DE. [Fig 8b(f)]. What do you observe?

Observations

Students will observe that

1. ∠BPA does not cover ∠DEF completely. [Fig 8b(f)]
2. ∠BPA is smaller that the ∠DEF.
3. ∠DEF is 90^0.
4. Therefore, ∠BPA is acute.
5. ∠BPA is an angle in the major segment.

Procedure for (c)

1. Draw a circle of any radius with centre O on a coloured sheet of paper and cut it.
2. Paste the cutout on a rectangular sheet of paper. [Fig 8c(a)]
3. Fold the circle in such a way that a chord AB is obtained. Draw AB. [Fig 8c(b)]
4. Take a point P in the minor segment.
5. Form a crease joining AP. Draw AP. [Fig 8c(c)]
6. Form a crease joining BP. Draw BP. [Fig 8c(d)]
7. Make a replica of ∠APB. [Fig 8c(e)]
8. Place the right angled Δ DEF on the replica of ∠APB such that DE falls on BP. [Fig 8c(f)] What do you observe?

Observations

Students will observe that

1. ∠ DEF does not cover ∠ BPA completely. [Fig 8c(f)]
2. ∠ DEF is smaller that the ∠ BPA.
3. ∠DEF is 90^0.
4. Therefore, ∠BPA is obtuse.
5. ∠BPA is an angle in the minor segment.

Learning outcome

Students develop familiarity with the fact that the angle in a semicircle is right angle, the angle in a major segment is acute angle and the angle in a minor segment is obtuse angle.

Remark

The teacher may point out that for a given chord AB, the obtuse angle in the minor segment and the acute angle in the major segment are supplementary angles. The students may be asked to verify this by taking appropriate cutouts of the angles.

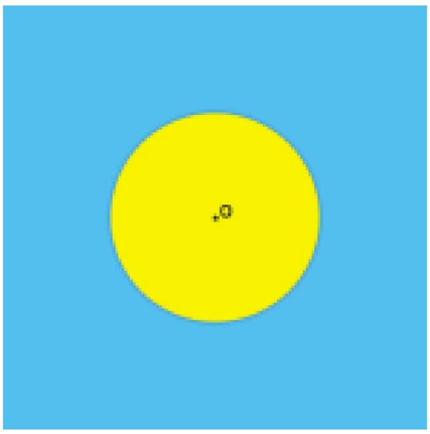


Fig 8a(a)

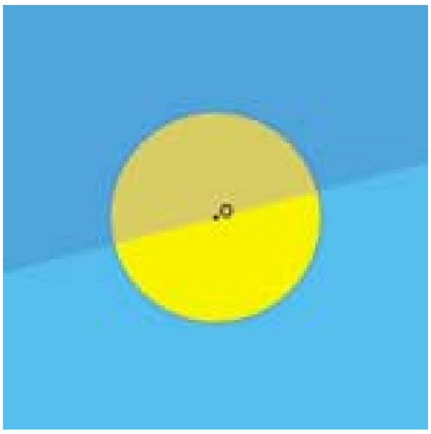


Fig 8a(b)

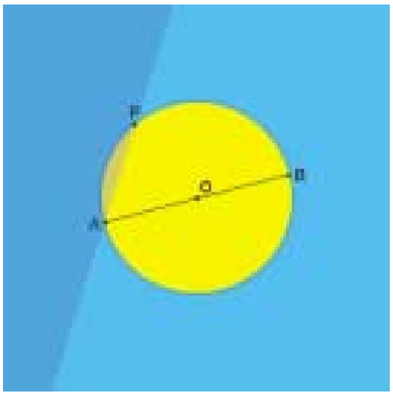


Fig 8a(c)

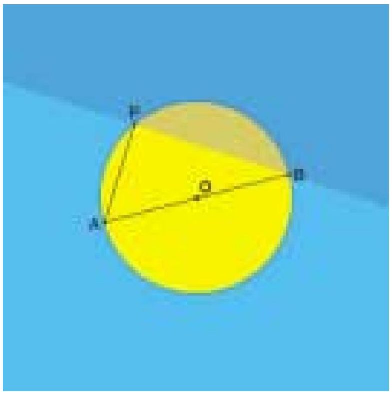


Fig 8a(d)

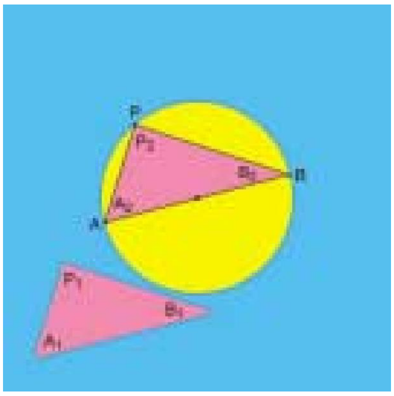


Fig 8a(e)

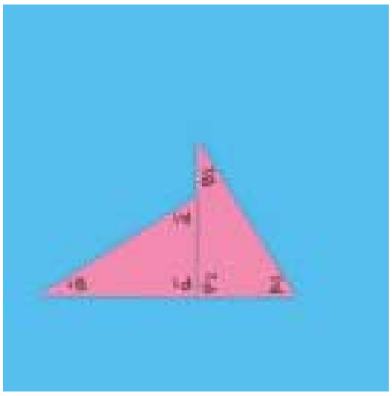


Fig 8a(f)

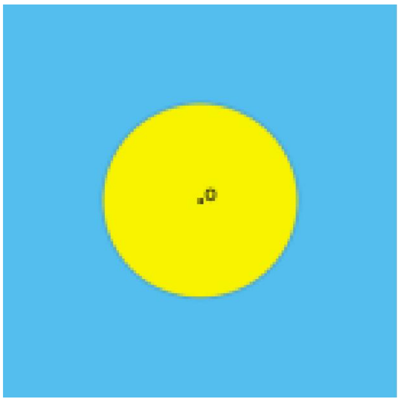


Fig 8b(a)

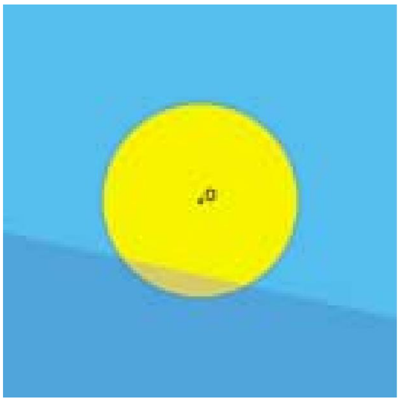


Fig 8b(b)

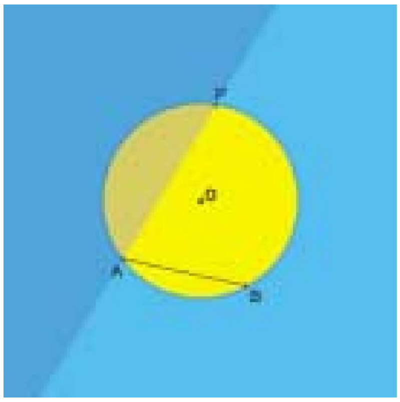
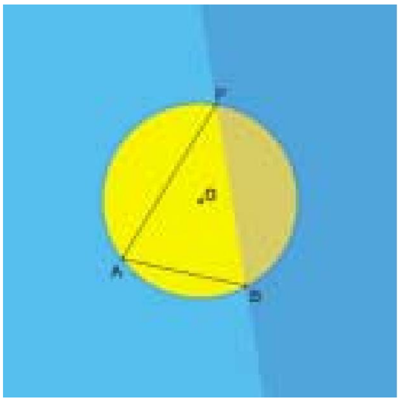


Fig 8b(c)



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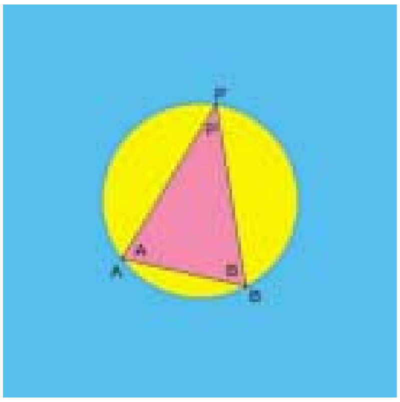

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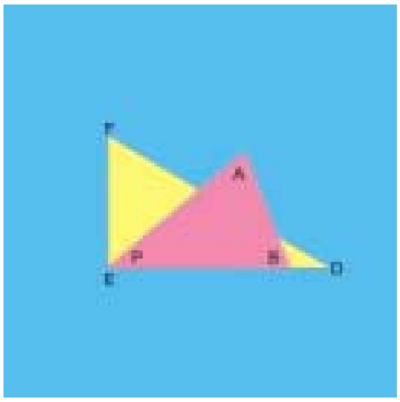

Fig 8b(f)

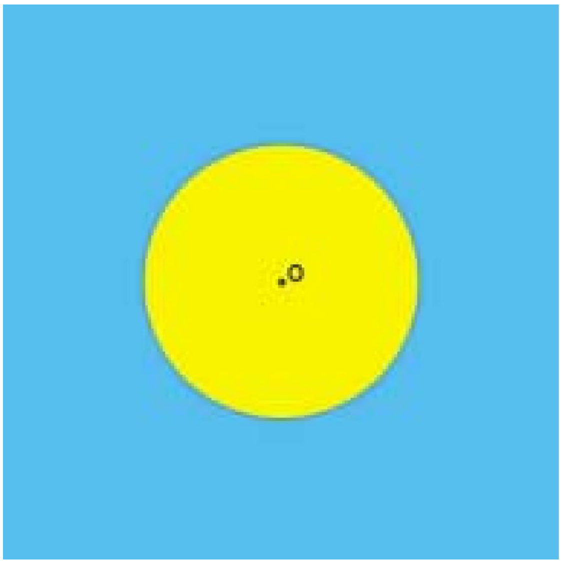

Fig 8c(a)

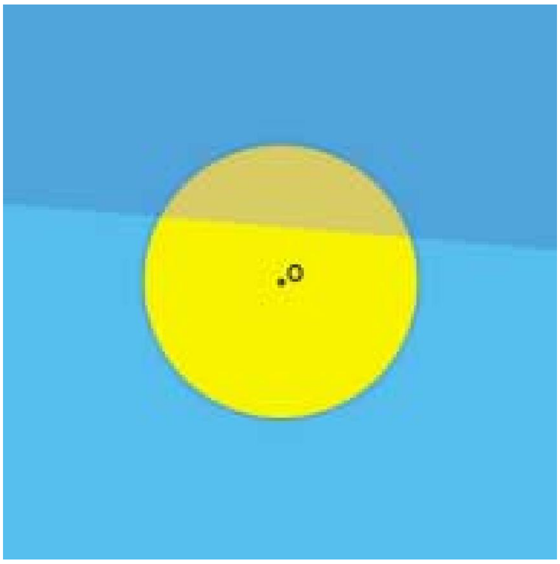

Fig 8c(b)

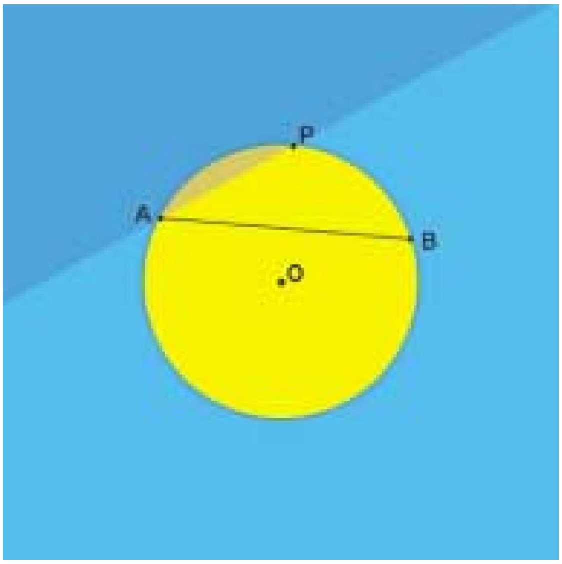

Fig 8c(c)

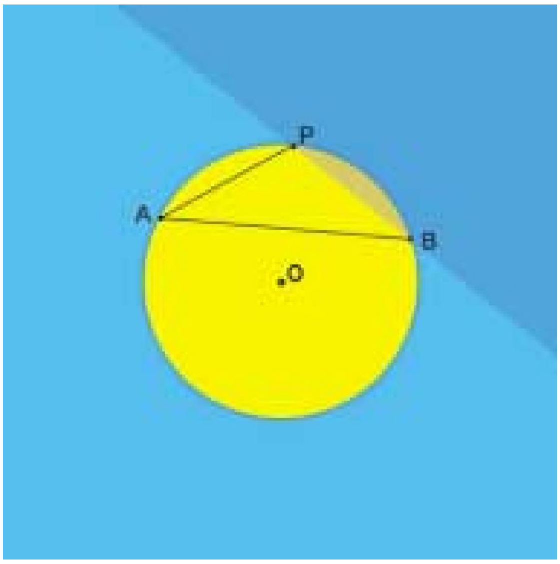

Fig 8c(d)

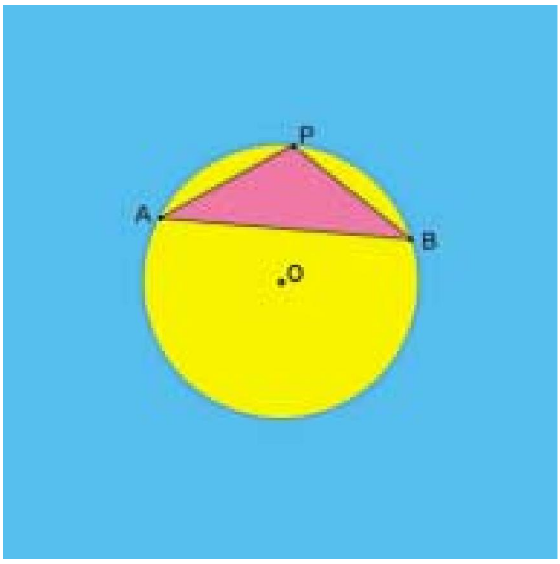

Fig 8c(e)

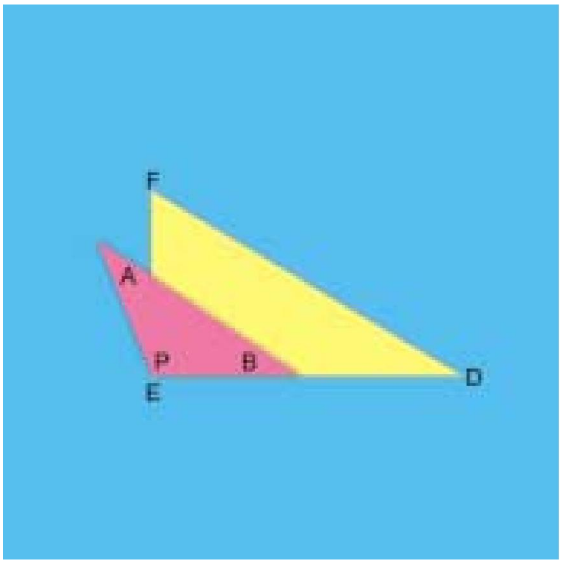

Fig 8c(f)

Activity 9

Cyclic Quadrilateral Theorem

Materials required

coloured papers,
pair of scissors,
ruler,
sketch pen,
carbon paper or
tracing paper.

Objective

To verify, using the method of paper cutting, pasting and folding that

1. the sum of either pair of opposite angles of a cyclic quadrilateral is 180^0.
2. in a cyclic quadrilateral the exterior angle is equal to the interior opposite angle.

Pre-requisite knowledge

1. Meaning of cyclic quadrilateral, interior opposite angle
2. Linear pair axiom

Procedure

1. Draw a circle of any radius on a coloured paper and cut it.
2. Paste the cutout on a rectangular sheet of paper.[Fig 9(a)].
3. By paper folding get chords AB, BC, CD and DA.
4. Draw AB, BC, CD and DA. Cyclic quadrilateral ABCD is obtained [Fig 9(b)].
5. Make a replica of cyclic quadrilateral ABCD using carbon paper / tracing paper. [Fig 9(c)]
6. Cut the quadrilateral cutout into four parts such that each part contains one angle i.e. $\angle$A, $\angle$B, $\angle$C and $\angle$D. [Fig 9(d)]
7. Place $\angle$A and $\angle$C adjacent to each other. What do you observe? [Fig 9(e)]
8. Produce AB to form a ray AE. Exterior angle $\angle$CBE is formed. [Fig 9(f)]
9. Place the replica of D on $\angle$CBE. [Fig 9(g)] What do you observe?

Observations

Students will observe that

1. When $\angle$A and $\angle$C are placed adjacent to each other they form a linear pair. This shows $\angle A + \angle C = 180^0$
2. $\angle$D completely covers $\angle$CBE. This shows that exterior angle of a cyclic quadrilateral ABCD is equal to the interior opposite angle.

Learning outcome

Students develop geometrical intuition of the result that

1. opposite angles of a cyclic quadrilateral are supplementary.
2. exterior angle to a cyclic quadrilateral is equal to the interior opposite angle.

Remarks

1. The teacher may ask the students to perform the activity for the other pair of angles (i.e. $\angle$B and $\angle$D) and for the other exterior angles also.
2. The teacher should point out that this theorem is true only for a cyclic quadrilateral. Students may be asked to perform a similar activity for a non-cyclic quadrilateral.

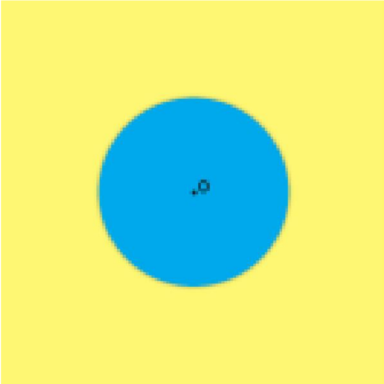

Fig 9(a)

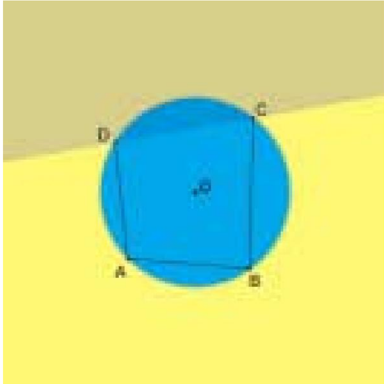

Fig 9(b)

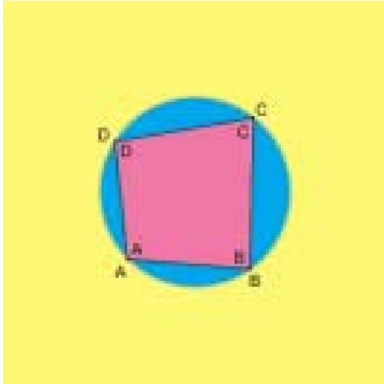

Fig 9(c)

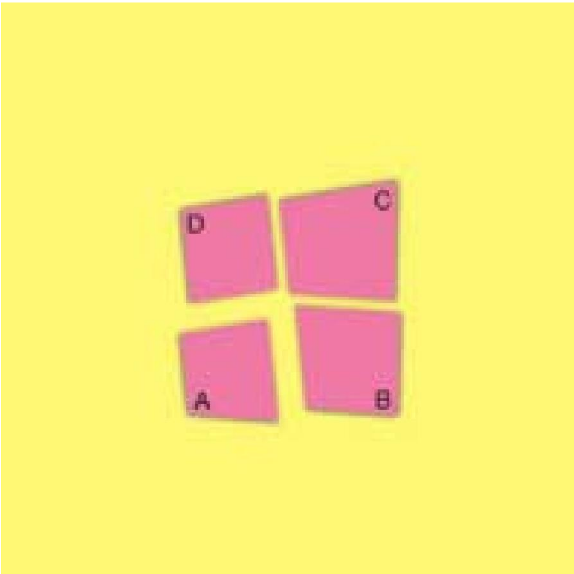

Fig 9(d)

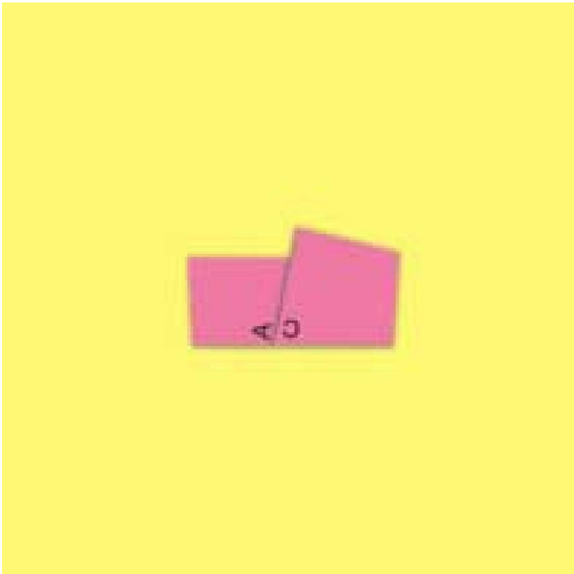

Fig 9(e)

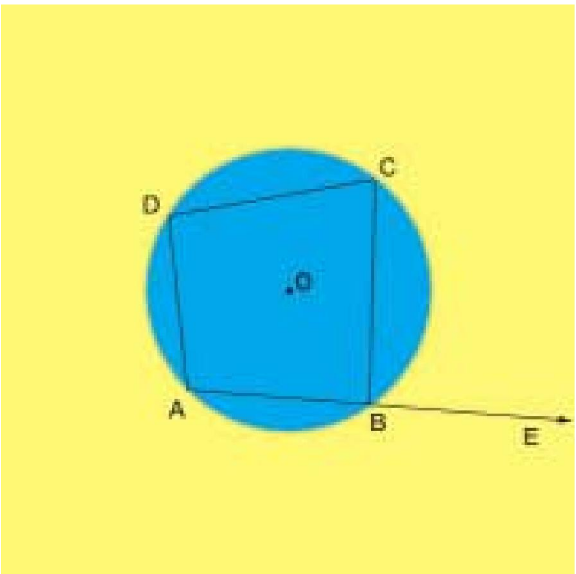

Fig 9(f)

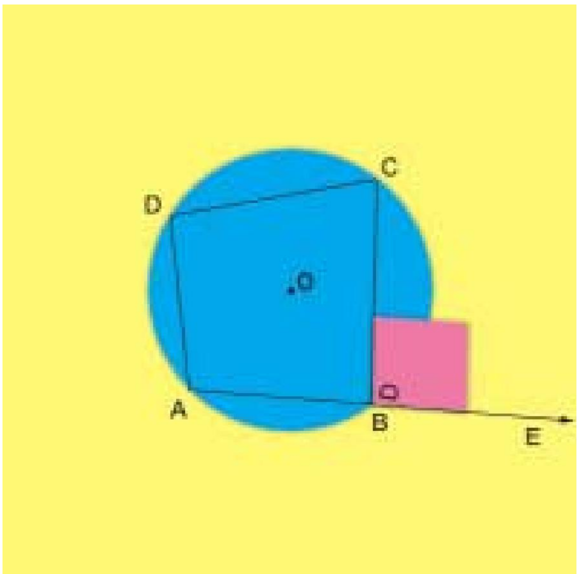

Fig 9(g)

Activity 10

Tangents drawn from an external point

Materials required

coloured papers,
pair of scissors,
ruler,
sketch pens,
compass,
pencil.

Objective

To verify using the method of paper cutting, pasting and folding that the lengths of tangents drawn from an external point are equal.

Pre-requisite knowledge

Meaning of tangent to a circle.

Procedure

1. Draw a circle of any radius on a coloured paper and cut it. Let O be its centre.
2. Paste the cutout on a rectangular sheet of paper.[Fig 10(a)]
3. Take any point P outside the circle.
4. From P fold the paper in such a way that it just touches the circle to get a tangent PA (A is the point of contact). [Fig 10(b)]. Join PA.
5. Repeat step 4 to get another tangent PB to the circle (B is the point of contact). [Fig 10(c)]. Join PB.
6. Join the centre of the circle O to P, A and B. [Fig 10(d & e)]
7. Fold the paper along OP. [Fig 10(f)] What do you observe?

Observations

Students will observe that

1. Δ OPA and Δ OPB completely cover each other.
2. Length of tangent PA = Length of tangent PB.

Learning outcome

Students learn how to get tangents from an external point to a circle using paper folding and verify the theorem.

Remark

The teacher may ask the students to perform the activity by taking point P (external point) at different locations.

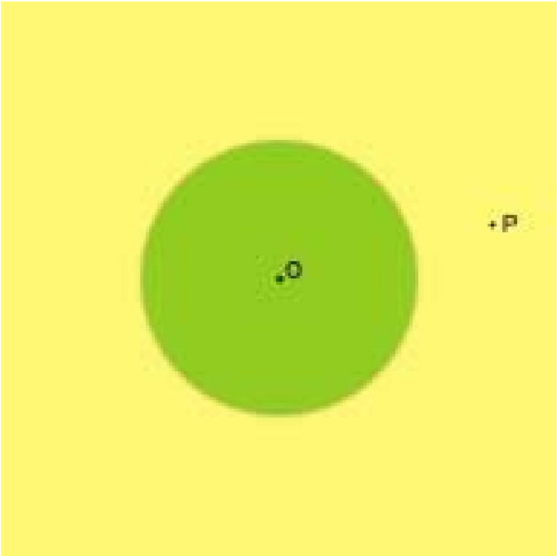

Fig 10(a)

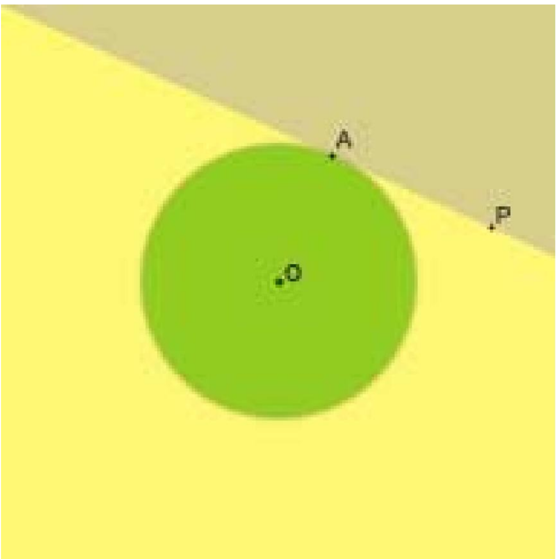

Fig 10(b)

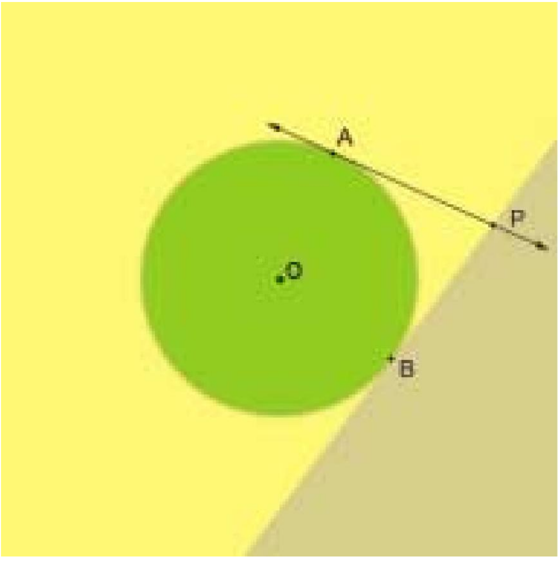

Fig 10(c)

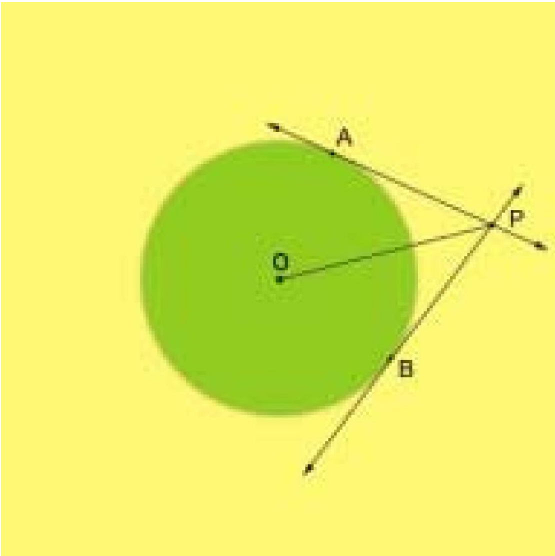

Fig 10(d)

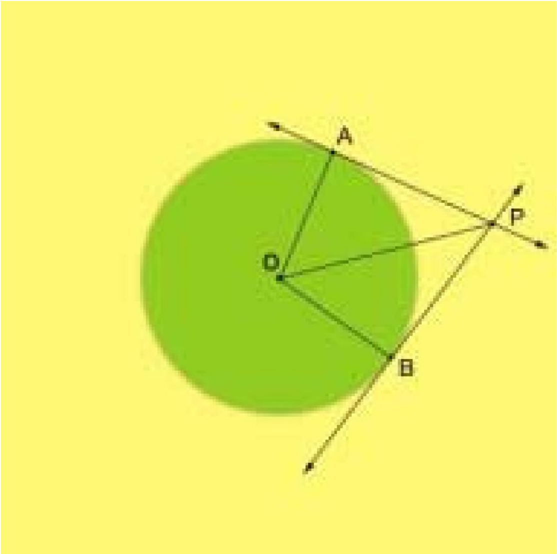

Fig 10(e)

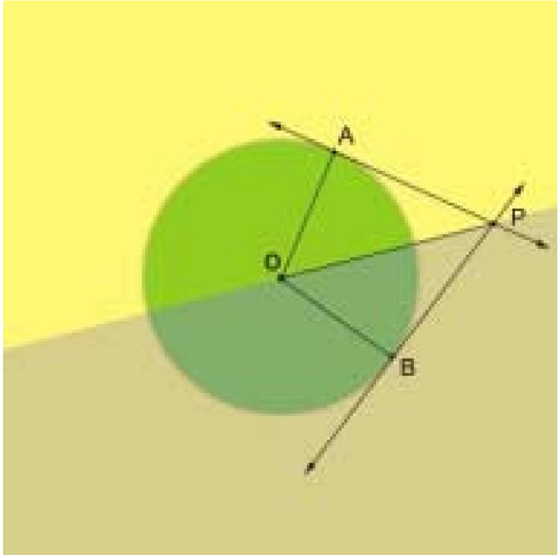

Fig 10(f)

Activity 11

Alternate Segment Theorem

Materials required

coloured paper,
pair of scissors,
ruler,
sketch pen,
fevistik,
carbon paper or
tracing paper.

Objective

To verify the Alternate Segment Theorem by the method of paper cutting, pasting and folding.

> ***Alternate Segment Theorem***
> *If a chord is drawn through the point of contact of a tangent to a circle, then the angles which this chord makes with the given tangent are equal respectively to the angles formed in the corresponding alternate segments.*

Pre-requisite knowledge

Geometrical terms related to a circle.

Procedure

1. Draw a circle of any radius on a coloured paper and cut it.
2. Paste the cut out on a rectangular sheet. [Fig 11(a)]
3. Fold the sheet of paper in such a way that it just touches the circle.[Fig 11(b)]
4. Unfold the paper and draw tangent PQ. Let A be the point of contact as shown in Fig 11(c).
5. Fold the paper starting from A such that chord AB is obtained. Draw AB. [Fig 11(d)].
6. Observe the angles formed by the chord AB and the tangent PQ: $\angle$BAP and $\angle$BAQ.
7. Take a point C on the major arc.
   Form a crease joining AC. Draw AC.[Fig 11(e)]
   Form a crease joining BC. Draw BC. [Fig 11(f)]
8. Take a point D on the minor arc.
   Form a crease joining AD. Draw AD.[Fig 11(g)]
   Form a crease joining BD. Draw BD. [Fig 11(h)]
9. Make a replica of $\angle$ACB, using a carbon / tracing paper. [Fig 11(i)]
   Place it on $\angle$BAQ. [Fig 11(j)]
   What do you observe?
10. Make a replica of $\angle$BDA. [Fig 11(k)]
    Place it on $\angle$BAP. [Fig 11(l)]
    What do you observe?

Observations

1. Students will observe that the chord AB is making two angles $\angle$BAQ and $\angle$BAP with the tangent PQ.
2. They will also observe that replica of $\angle$ACB completely covers $\angle$BAQ and replica of $\angle$ADB completely covers $\angle$BAP. They will then verify the theorem.

Learning outcome

Students will enhance their familiarity with the Alternate Segment Theorem through an activity.

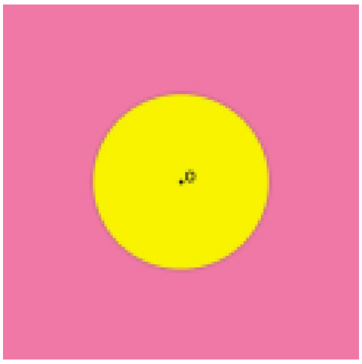

Fig 11(a)

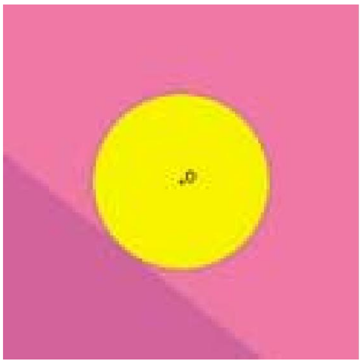

Fig 11(b)

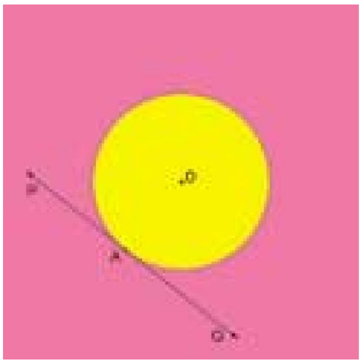

Fig 11(c)

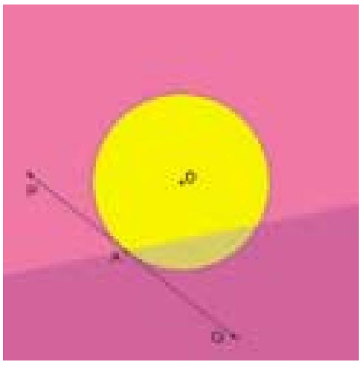

Fig 11(d)

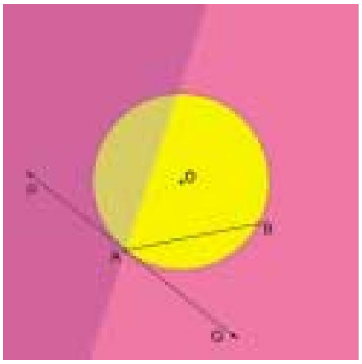

Fig 11(e)

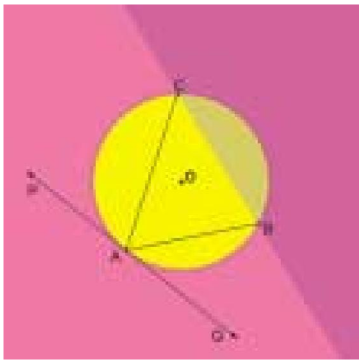

Fig 11(f)

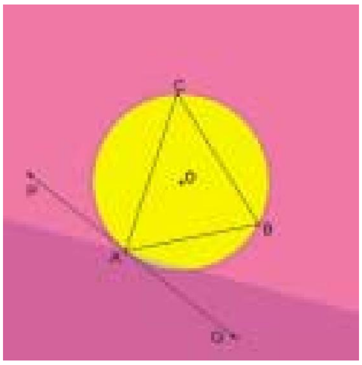

Fig 11(g)

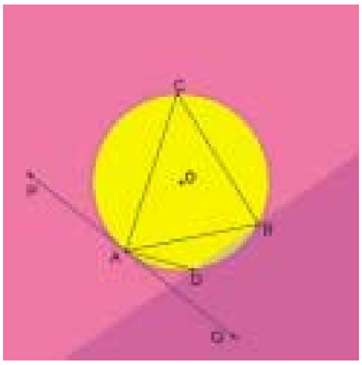

Fig 11(h)

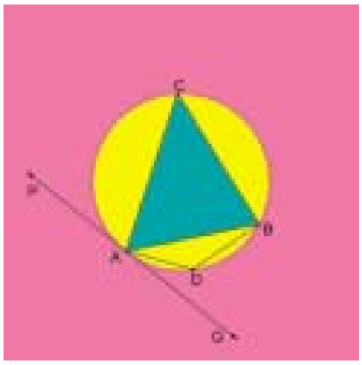

Fig 11(i)

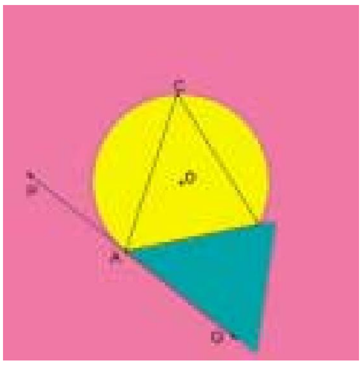

Fig 11(j)

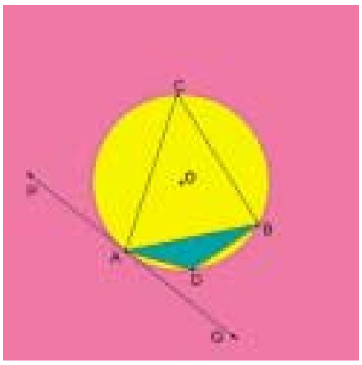

Fig 11(k)

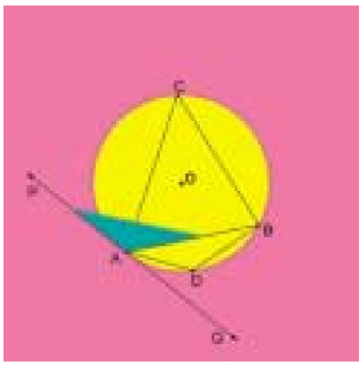

Fig 11(l)

Activity 12

Right circular cylinder

> **Materials required**
>
> coloured papers, a pair of scissors, gum.

Objective

To make a right circular cylinder of given height and circumference of base.

Pre-requisite knowledge

1. Drawing and cutting a rectangle of given dimensions.
2. Formula for the circumference of a circle.

Procedure

1. Cut a rectangular sheet of paper of length l = given circumference of base of cylinder, breadth b = given height. [Fig 12(a)].
2. Gently curve the paper so that the two (shorter) sides come together.
3. Join the edges together by cello tape.[Fig 12(b)].

Observations

1. The rectangle transforms into a cylinder.
2. The height of the cylinder is b.
3. The circumference of the base circle is l.

Learning outcomes

1. Students learn to make a cylinder of given height and base circumference.
2. Students appreciate how folding of geometrical figures transforms their shape.

Remark

The teacher may suggest to students that b and l may be interchanged to form a different cylinder.

Fig 12(a)

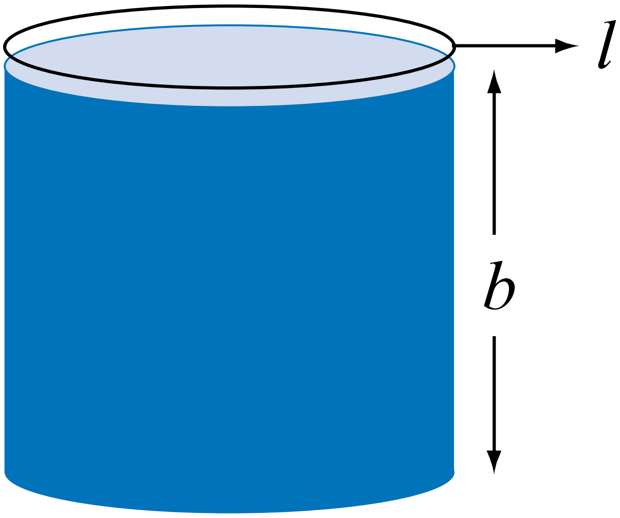

Fig 12(b)

Activity 13

Surface area of a cylinder

Materials required

cylinder of known dimension made of chartpaper,
pair of scissors,
gum,
ruler.

Objective

1. To determine the area of a given cylinder.
2. To obtain the formula for the lateral surface area of a right circular cylinder in terms of the radius (r) of its base and height (h).

Pre-requisite knowledge

1. A rectangle can be rolled to form a cylinder.
2. Area of a rectangle.
3. Area of a circle.

Procedure

1. Remove the top and bottom circles of the cylinder. [Fig 13(b)]
2. Make a vertical cut in the curved surface and lay the cylinder flat. [Fig 13(b)]
3. Measure the length and breadth of the rectangle so formed.

Observations

1. The base and top of the cylinder are congruent circular regions.
2. The curved surface area of the cylinder opens to form a rectangular region.
3. The breadth of the rectangle is the height of the cylinder.
4. The length of the rectangle is the circumference of the base of the cylinder.
5. Curved surface area of cylinder (c) = area of rectangle

   $c = l \times b$

   Since $l = 2\pi r$

   $b = h$

   $c = l \times b = 2\pi rh$
6. Total surface area of cylinder = curved surface area (c) + 2 (area of base circle)

   $= 2\pi rh + 2\pi r^2$

   $= 2\pi r\,(h + r)$

Learning outcome

Students appreciate the derivation of the formula for the curved surface area of a right circular cylinder.

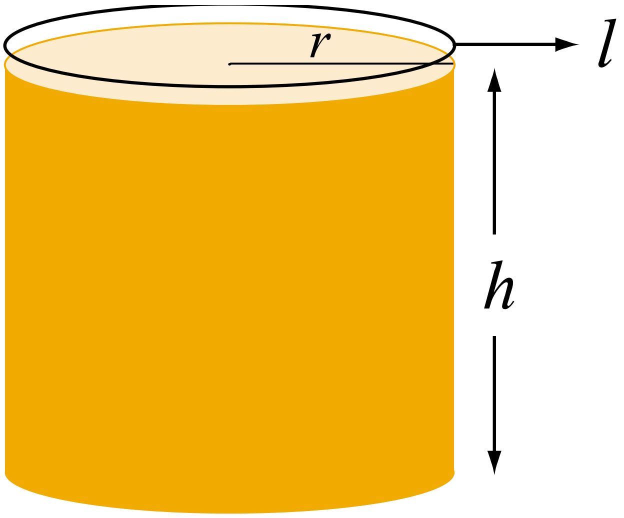

Fig 13(a)

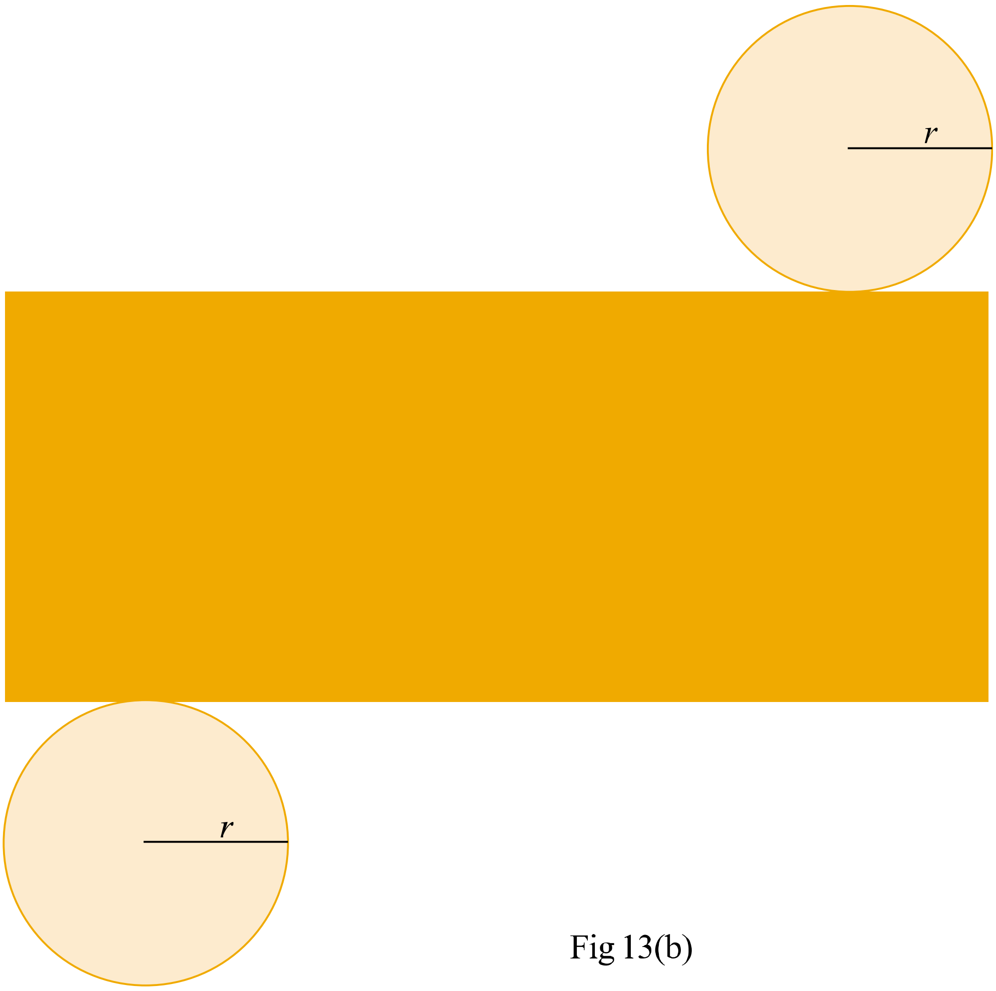

Fig 13(b)

Activity 14

Volume of a right circular cylinder

Materials required

Thermacol, plastic clay.

Objective

To give a suggestive demonstration of the formula for the volume of a right circular cylinder in terms of its height and radius of the base circle.

Pre-requisite knowledge

1. Formula for volume of a cuboid
2. Formula for circumference of a circle.

Procedure

1. Make a cylinder of any dimensions using plastic clay. Let its height be h and radius of base circle r.
2. Cut the cylinder into 8 sectorial sections as shown in the figure. [Fig 14(a)].
3. Place the segments alternately as shown in the figure. [Fig 14(b)].

Observations

The students observe that

1. The segments approximately form a solid cuboid of height 'h', breadth 'r' and length 'πr'.
2. The volume of the cuboid is $lbh = \pi r \times r \times h = \pi r^2 h$

Learning outcome

Students learn that the volume of a cylinder is $\pi r^2 h$ where r is the radius of the base and h the height of the cylinder.

Remarks

1. The teacher may help the student observe that the length of the cuboid is half the circumference of the base of the cylinder.
2. The teacher should point out that this activity does not give an exact proof of the formula and that the approximation improves by increasing the number of sectorial sections.

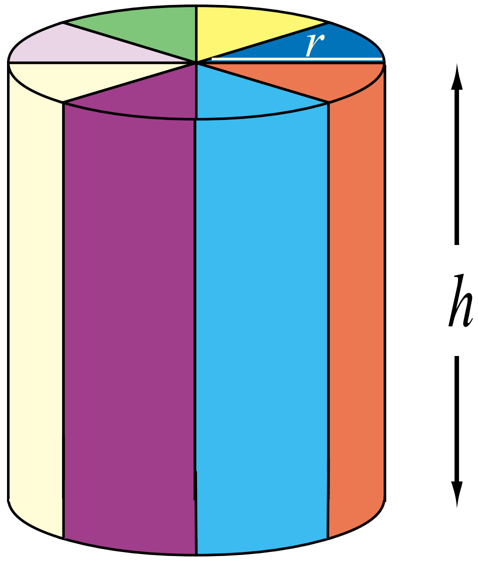

Fig 14(a)

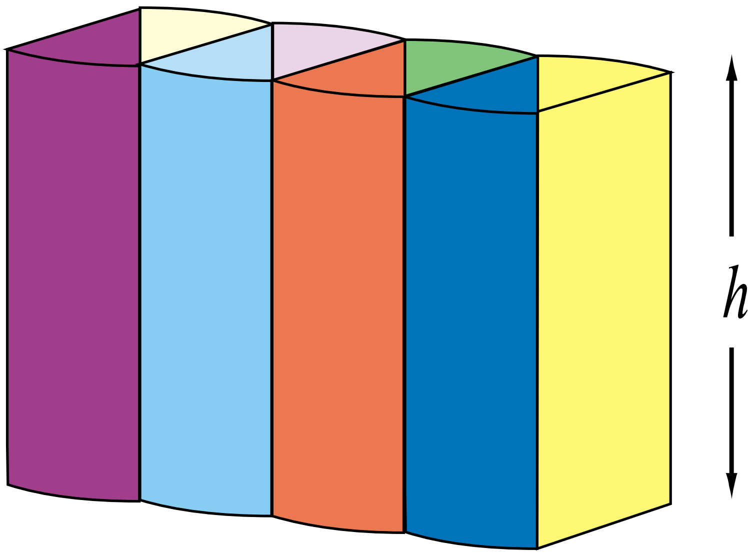

Fig 14(b)

Activity 15

Right circular cone

Materials required

chart paper,
a pair of scissors,
gum,
scale.

Objective

To make a cone of given slant length l and base circumference.

Pre-requisite knowledge

1. Circumference of a circle.
2. Sector of a circle.
3. Pythagoras theorem.

Procedure

1. Draw a circle with radius equal to the slant height l. [Fig 15(a)].
2. Mark a sector OAB such that arc length A × B equals the given base circumference of the cone. [Fig 15(a)].
3. Cut the sector AOB and gently fold, bringing the 2 radii OA and OB together. [Fig 15(b)]

Observations

1. Students will observe that when the radii of the sector are joined, a cone is formed.
2. The radius of the circle becomes the slant length of the cone.
3. The arc length becomes the circumference of the base of the cone.

Learning outcome

The students learn how to make a cone of given slant height and base circumference from a sector of a circle.

Remark

The teacher may ask student to determine the radius and the height of the cone formed, using the formula for the circumference of a circle and Pythagoras theorem.

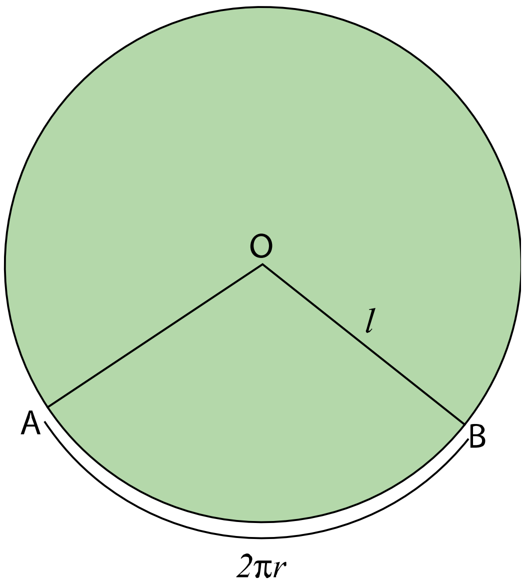

Fig 15(a)

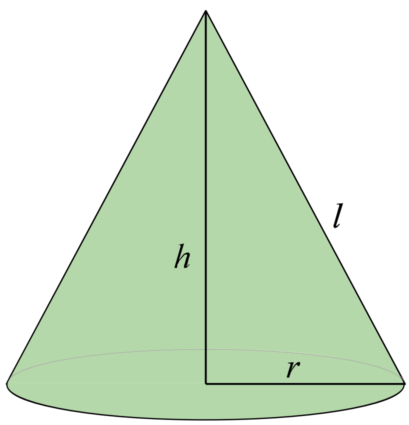

Fig 15(b)

Activity 16

Surface area of a cone

Materials required

chart papers,
a pair of scissors,
gum.

Objective

To give a suggestive demonstration of the formula for the lateral surface area of a cone.

Pre-requisite knowledge

1. The lateral surface of a cone can be formed from a sector of a circle.
2. Formula for area of a parallelogram.

Procedure

1. Cut vertically and unroll the cone. Identify the region. The region is a sector of circle. [Fig 16(a & b)]
2. Identify the arc length of the sector as the base circumference of the cone and the radius of the sector as the slant height of the cone.
3. Fold and cut the sector into 4 (even number of) equal smaller sectors. [Fig 16(b)]
4. Arrange the smaller sectors to form approximately a parallelogram. [Fig 16(c)]

Observations

1. Students observe that the base of the parallelogram is roughly half the circumference of the base of the cone. i.e. $\frac{1}{2} \times 2\pi r = \pi r$.
2. The height of the parallelogram is roughly the slant height of the cone 'l'.
3. Therefore, curved surface area = area of the parallelogram = πrl.

Learning outcome

1. Students learn that the surface area of a cone is πrl where r is the radius of the cone and l is the slant height.
2. Students appreciate how folding turns a plane surface (sector of a circle) into a curved surface (of the cone), and vice versa.

Remark

1. The teacher may help students observe that the base of the parallelogram is half the base circumference of the cone.
2. The teacher should point out that this activity does not give an exact proof of the formula, and the approximation improves by increasing the number of divisions of the sector.
3. The teacher may point out that the total surface area of the cone may be obtained by adding curved surface area to the area of the base.
i.e. total area = $\pi rl + \pi r^2$

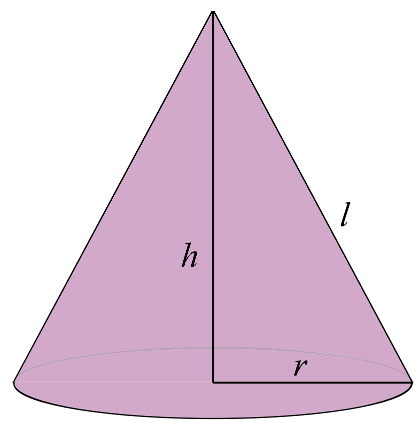

Fig 16(a)

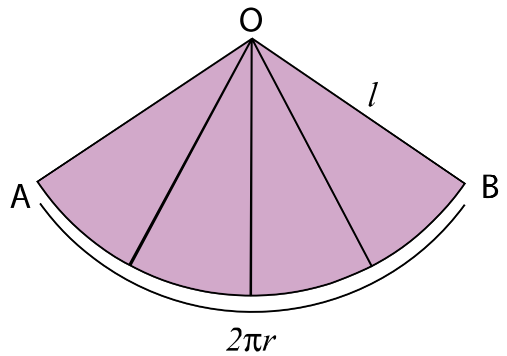

Fig 16(b)

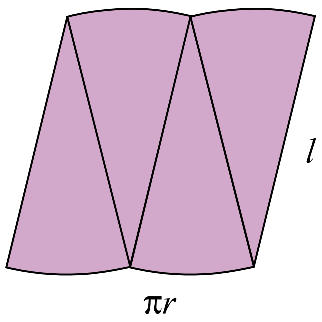

Fig 16(c)

Activity 17

Volume of a cone

> **Materials required**
>
> 3 sets of a cone and cylinder. In each set, the cone and cylinder have the same height and base radius.

Objective

To give a suggestive demonstration of the formula for the volume of a right circular cone.

Pre-requisite knowledge

1. Concept of volume and its proportionality to quantity of matter.
2. Formula for the volume of a cylinder.

Procedure

1. Take one set of the cone and cylinder.
2. Fill the cone with sand.
3. Pour the sand from the cone to the cylinder.
4. Fill the cone again with sand and repeat step 3 to fill the cylinder completely with sand.
5. Repeat the activity with other sets of cones.

Observation

The student observes that for each set, it needs three pourings from the cone to fill the cylinder completely.

Learning outcomes

1. The volume of a cone is one-third the volume of the cylinder of the same height (h) and radius of base (r), i.e. equal to $= \frac{1}{3}\pi r^2 h$
2. Because of the simplicity of the concrete activity and the ratio (3) involved, students are likely to remember the result of the activity easily.

Remark

1. The teacher should see that the students fill the cone with sand properly.
2. The teacher should note that the activity makes use of the proportionality between volume and quantity of matter.

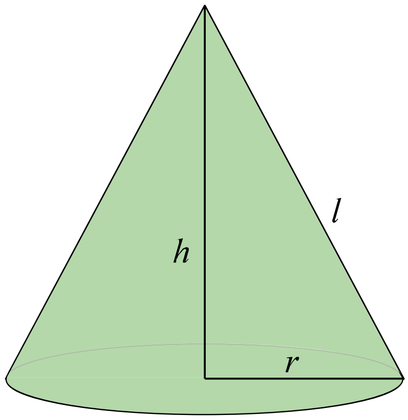

Fig 17(a)

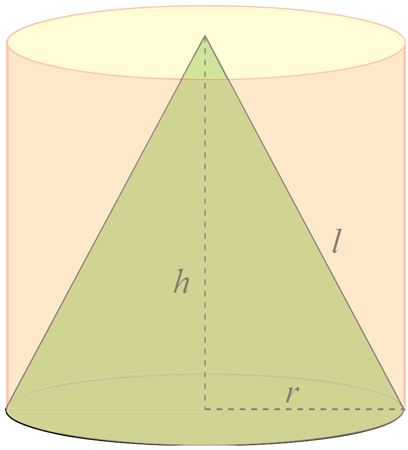

Fig 17(b)

Activity 18

Surface area of a sphere

Materials required

Hollow sphere cut into two hemispheres, a cylinder with both base diameter and height equal to the diameter of the sphere

Objective

To give a suggestive demonstration of the formula for the surface area of a sphere in terms of its radius.

Pre-requisite knowledge

Curved surface area of cylinder = $2\pi rh$

Procedure

1. Take a roll of a jute thread and wind it closely on the surface of the hemisphere completely. [Fig 18(a)].
2. Take another roll of jute thread and wind it completely along the curved surface of the cylinder. [Fig 18(b)].
3. Compare the length of the two threads.

Observations

1. Students observe that the length of the thread used to cover the curved surface of the cylinder is twice the length needed to cover the hemisphere.
2. Since the thickness of the thread is uniform and the same for both the threads, surface areas are proportional to the lengths of the threads approximately.
3. Hence surface area of the hemisphere = half the surface area of the cylinder

$$= \frac{1}{2} \times 2\pi rh$$
$$= \pi rh$$
$$= \pi r \times 2r \quad (\because h = 2r)$$
$$= 2\pi r^2$$

Therefore, surface area of a sphere = $4\pi r^2$

Learning outcome

The student arrives at the formula for the surface area of a sphere through a simple activity, which relates it to the area of a cylinder.

Remark

The teacher should point out that this activity is not an exact proof of the formula, but is only a simple but approximate approach to appreciate the formula. The approximation improves with a thinner thread and tight and uniform winding.

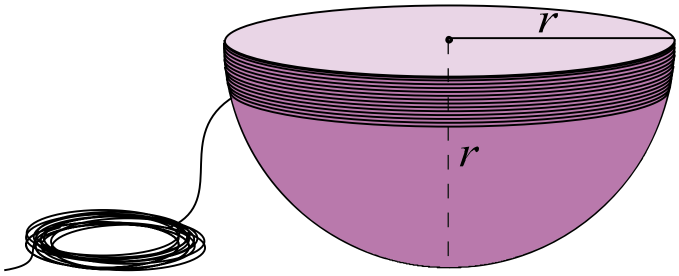

Fig 18(a)

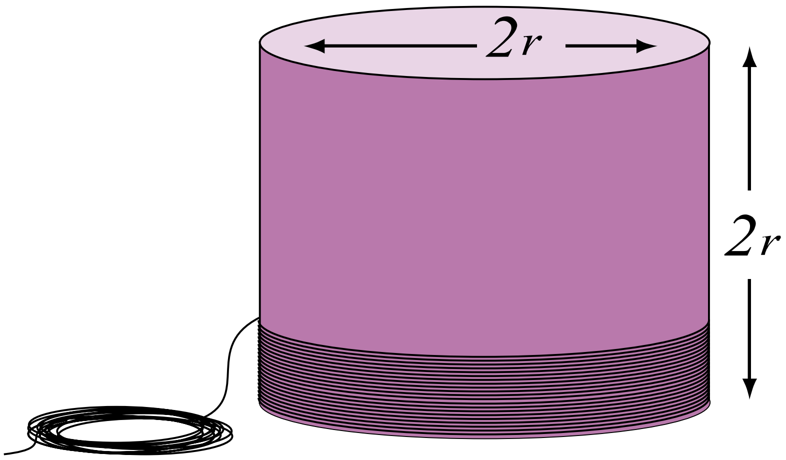

Fig 18(b)

Activity: 19

Volume of a sphere

Objective

To give a suggestive demonstration of the formula for the volume of a sphere in terms of its radius.

Pre-requisite knowledge

Volume of a cylinder.

Materials required

A hollow sphere and two cylinders whose base diameter and height are equal to the diameter of the sphere, sand.

Procedure

1. Fill the hollow sphere with sand once and empty it into one of the cylinders.
2. Fill the hollow sphere a second time with sand and empty it into the second cylinder.
3. Fill the hollow sphere a third time and empty it into the remaining spaces of the two cylinders.

Observations

1. Students observe that the total sand emptied in three pourings fill both the cylinders completely.
2. They, therefore, conclude that

3 times the volume of sphere $= 2$ times the volume of cylinder $= 2\pi r^2 h$

$= 4\pi r^3 \quad (\because h = 2r)$

$\therefore$ Volume of sphere $= \dfrac{4}{3}\pi r^3$

Learning outcome

The students arrive at the formula for the volume of a sphere through a simple activity, which relates it to the volume of a cylinder.

Remarks

1. The teacher should point out that this activity is not an exact proof of the formula, but is only a simple but approximate approach to appreciate the formula. The approximation improves with the use of suitable materials (in place of sand) that do not leave air gaps.

2. The teacher can advise the students to try the activity with other suitable materials.

3. The teacher should note that the activity makes use of the proportionality between the volume and quantity of matter.

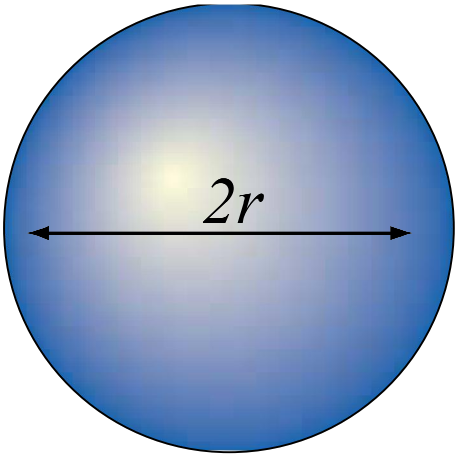

Fig 19(a)

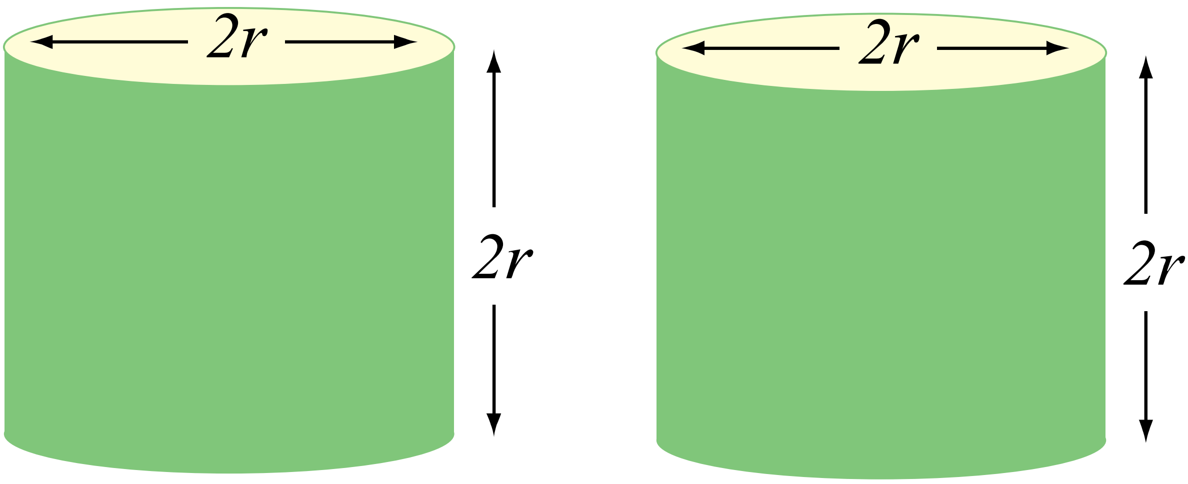

Fig 19(b)

Activity 20

Finding probability

Materials required

card board of size 15 cm × 15 cm, glazed paper (2 colours), pair of scissors, fevistick, sketch pens and an empty box.

Objective

To get familiar with the idea of probability of an event through a double colour card experiment.

Pre-requisite knowledge

The formula of probability of an event E is : P (E) = No. of favorable outcomes to E / Total no. of outcomes.

Procedure

A. Preparation of material for performing the activity.

1. Take a card board and paste glazed papers of different colours on both sides. (Say red and yellow.) [Fig 20(a)]
2. Cut the cardboard into 36 small squared cards.
3. Write all the 36 possible outcomes obtained by throwing two dice. [Fig 20(b)]. e.g. for the outcome (2, 1), write 2 on the yellow side and 1 on the red side of the squared card.
4. Put all the cards into a box.

B. For finding the required probability of an event do the following

1. Take out each card one by one without replacement and fill the observation table by putting (√) on favorable outcomes and (×) otherwise.
2. Count the total number of total possible outcomes from column 2. Write total possible outcomes.
3. Count the (√) marks from the columns 3, 4, 5 and 6.

Sr. No.	Possible outcomes		Sum ≥ 9	Sum < 5	Sum = 7	Odd on yellow & even on red.
	yellow card	red card				
1.	1	3	×	√	×	×
2.						
………………………………………						
………………………………………						
………………………………………						
36.						

Observations

Total number of possible outcomes = ______________________________

Total number of favorable outcomes (Sum ≥ 9) = ____________________________

Total number of favorable outcomes (Sum < 5) = ______________________________

Total number of favorable outcomes (Sum = 7) = ______________________________

Total number of favorable outcomes (even number on one side of the card and odd on other) = ______________________________

Using the formula calculate the required probability of each event.

Remark

In this experiment, the student does not put back the card after taking it out. Consequently, the number of favourable outcomes for any event is certain. To arrive at the true notion of probability, the card should be put back and the experiment repeated a very large number of times. This, however, may be impractical in the actual classroom situation.

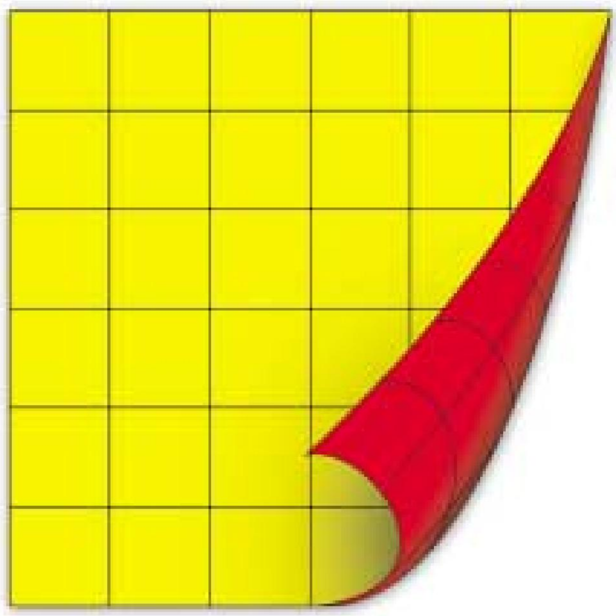

Fig 20(a)

(1,1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
(5,1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
(6,1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

Fig 20(b)